OPERATIONS WITH RADICALS (SQUARE ROOTS)

Division

To work with division of square roots, you use the fact that $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

For this answer to be in simplest form, there must be no square root sign on the bottom of the fraction. Following are some examples:

$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$ $\sqrt{\frac{16}{36}} = \frac{\sqrt{16}}{\sqrt{36}} = \frac{4}{6}$ which, then, reduces to $\frac{2}{3}$

$\sqrt{\frac{3}{5}}$ Since the number on the bottom is not a perfect square, we mult. the fraction time $\frac{5}{5}$ so that the bottom will be a pefect square.

$\sqrt{\frac{3}{5}} = \sqrt{\frac{15}{25}} = \frac{\sqrt{15}}{5}$

$\sqrt{\frac{1}{8}} = \sqrt{\frac{2}{16}}$ (mult. top + bottom times 2) $= \frac{\sqrt{2}}{\sqrt{16}} = \frac{\sqrt{2}}{4}$

Addition/Subtraction

In many ways, working with radicals is like working with variables: you can only combine like radicals. If the radicals are not "like", one cannot simplify the answer. See the examples below:

$\sqrt{3} + \sqrt{5}$ – Cannot be simplified

$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$ (add/subtract number in front; radical stays the same)

$5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

$\sqrt{45} + \sqrt{20}$ - can't be added but can be simplified:

$\sqrt{45} + \sqrt{20} = \sqrt{9 \cdot 5} + \sqrt{4 \cdot 5} = 3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

now you have like radicals!

Multiplying numbers with radicals

If the numbers are monomials, use the fact that $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ See examples below.

$\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

$\sqrt{5} \cdot \sqrt{11} = \sqrt{55}$

$\sqrt{6} \cdot \sqrt{3} = \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$

$\sqrt{3} \cdot \sqrt{3} = \sqrt{9} = 3$

$\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$

when you mult. a radical times itself, you get a whole number.

If they are binomials, FOIL them. See examples below:

$(2+\sqrt{5})(3+\sqrt{5}) = 6 + 2\sqrt{5} + 3\sqrt{5} + \sqrt{5} \cdot \sqrt{5}$

$= 6 + 5\sqrt{5} + 5$

$= 11 + 5\sqrt{5}$

$(3+\sqrt{7})(3-\sqrt{7}) = 9 - 3\sqrt{7} + 3\sqrt{7} - \sqrt{7} \cdot \sqrt{7}$

$= 9 + 0 - 7$

$= 2$